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Heat transfer enhancements in heat exchangers fitted with porous media Part I: constant wall temperature

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Abstract

This work investigates heat transfer enhancement for a flow in a pipe or a channel fully or partially filled with porous medium. The porous layer inserted at the core of the conduit. Forced, laminar flow is assumed and the effects of porous layer thickness on the rate of heat transfer and pressure drop were investigated. The Darcy number (permeability) is varied in the range of 10^{-6} to 10.0. Developing and fully developed flow conditions are considered in the analysis. It is found that the plug flow assumption is not valid for $Da > 10^{-3}$. The effect of varying the inertia term (Forchheimer term) is also investigated and it is found that the inertia term is not that important for $Da < 10^{-4}$ for the range of the parameters investigated. Partially filling the conduit with porous medium has two advantages: it enhances the rate of heat transfer, and the pressure drop is much less than that for a conduit fully filled with a porous medium.

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1. Introduction

Heat transfer enhancements in heat transfer devices, such as heat exchangers, have been studied extensively. Webb [1] reviewed and discussed different techniques used for heat transfer enhancements for single and multiphase flows such as vortex generators and mixers. On the other hand, using porous medium to enhance the rate of heat and mass transfer in energy systems has many advantages. The values of Nusselt number are approximately 50% higher than the values predicted for laminar flows in channels without porous materials [2]. Moreover, the convective heat transfer coefficient is higher for systems filled with porous material than the systems without porous material due to the high thermal conductivity of the porous matrix compared with the fluid thermal conductivity, especially for gas flows.

Porous media have been used for different industrial and geophysical applications and yet there are potentials to explore other applications by utilizing porous media, especially for energy systems, such as compact heat exchangers, heat pipes, electronic cooling and solar collectors. For some applications there is no need to fill completely the system with the porous medium, as a partial filling of the

porous medium is sufficient. Partial filling has advantage of reducing the pressure drop compared with a system filled completely with porous medium. Moreover, partial filling eliminates contact between the porous material and surface, which decreases heat losses from the porous material to the surface. Such a criterion is required in a system where the main purpose is to enhance the thermal coupling between the porous medium and fluid flow, and to eliminate strong thermal coupling between the system and the ambient. For instance, in the solar air heater developed by Mohamad [4], the main idea was to enhance the rate of heat transfer from the porous medium, which is heated by solar radiation, to air, and at the same time to reduce the heat losses to the ambient. Furthermore, partial filling can reduce pressure drop. A partial filling of a channel with porous media forces the flow to escape from the core region, depending on the permeability of the medium, to the outer region, which reduces the boundary layer thickness and consequently enhances the rate of heat transfer. The porous medium also modifies the effective thermal conductivity and heat capacity of the flow, and the solid matrix enhances the rate of radiative heat transfer in a system where the gas is the working fluid. Hence, the heat transfer enhancements take place by three mechanisms: flow redistribution, thermal conductivity modification and radiative property modification of the medium.

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Nomenclature

c	specific heat	$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}$	R_r	ratio of the porous medium radius to the pipe radius, or the porous layer thickness to the channel height = r_p/r_o , H_p/H
D	pipe diameter	m	T	temperature
Da	Darcy number = K/r_o^2 or $K/(H/2)^2$		u	velocity component in z -direction
F	inertia coefficient		U	dimensionless velocity in z -direction = u/u_{in}
Gz	Graetz number = $4PrRe/Z$		v	velocity component in r -direction
H	channel height	m	V	dimensionless velocity in r -direction = v/u_{in}
h	heat transfer coefficient	$\text{W}\cdot\text{m}^{-2}\cdot\text{K}$	$ u $	velocity magnitude = $(u^2 + v^2)^{1/2}$
h_v	volumetric heat transfer coefficient	$\text{W}\cdot\text{m}^{-3}\cdot\text{K}$	$ U $	dimensionless velocity magnitude = $(U^2 + V^2)^{1/2}$
k	thermal conductivity	$\text{W}\cdot\text{m}^{-1}\cdot\text{K}$	z	axial coordinate
K	permeability	m^2	Z	dimensionless = z/r_o or $z/H/2$
m	mass flow rate	$\text{kg}\cdot\text{s}^{-1}$	<i>Greek symbols</i>	
Nu	Nusselt number		μ	viscosity
p	pressure	Pa	θ	dimensionless temperature = $(T - T_{in})/(T_w - T_{in})$
P	dimensionless pressure = $p/(\rho u_{in}^2)$		ρ	density
Pe	Peclet number = $PrRa$		<i>Subscripts</i>	
Pr	Prandtl number		e	effective
r	radial coordinate	m	f	fluid
R	dimensionless radius = r/r_o		in	inlet condition
Re	Reynolds number = $\rho u_{in} r_o / \mu$ or $\rho u_{in} (H/2) / \mu$		m	bulk
Re_D	Reynolds number based on pipe diameter = $\rho u_{in} D / \mu$		w	wall
Re_H	Reynolds number based on the channel height = $\rho u_{in} H / \mu$			
r_p	porous matrix radius	m		
r_o	outer radius of the cylinder	m		

Beavers and Joseph [3] presented an empirical correlation for the velocity gradient at the clear fluid/porous interface, which assumes a velocity gradient at the interface proportional to the difference between the Darcian and the slip velocity. Vafai and Thiyagaraja [5] solved same problem analytically by using asymptotic expansion technique. An exact solution of the problem is presented by Vafia and Kim [6] for a fully developed flow over a flat plate. Kaviany [7] considered laminar developing flow through a porous layer sandwiched between isothermal parallel plates. Forced convection in a channel whose walls are layered by a porous medium was considered by Poulikakos and Kazmierczak [8] for constant heat flux and constant wall temperature conditions, both analytically and numerically. A numerical study was presented by Jang and Chen [9] for a forced flow in a parallel channel partially filled with a porous medium by adopting the Darcy–Brinkman–Forchheimer model with a thermal dispersion term. Chikh et al. [10] and [11] presented an analytical solution for the fully developed flow in annulus configuration partially filled with porous medium. Al-Nimr and Alkam [12] extended the analysis to the transient solution for annulus flow with porous layer. Recently, Alkam et al. [13] and Abu-Hijleh and Al-Nimr [14] studied transient forced convection behavior of a flow in parallel plate chan-

nels with a porous substrate attached to the one of the plates. Recent reviews of the subject are available in [2] and [15].

In the present work, steady laminar flow in a conduit fully or partially filled with a porous layer was considered numerically for constant wall temperature boundary conditions. The constant wall temperature condition is encountered in many industrial heat exchanger applications, such as the condensation or evaporation of fluid on the outer surface of conduits. Also, if the heat transfer coefficient on the outer surface is higher than that on the inner surface, the assumption of a constant wall temperature can be justified.

Simulations are performed for flow in pipes and channels filled with porous medium at the core of the conduit. The results are presented for different permeabilities and porous layer thicknesses. The main objective of the this work is to analyze heat transfer enhancement that can be achievable and pressure drop due to fully or partially filling a conduit (pipe or channel) with porous medium. The present results show that the assumption of plug flow is not valid for $Da > 10^{-3}$. The inertia effects are significant in the same range of Darcy number, while inertia terms may be neglected for $Da < 10^{-4}$, at least for the range of parameters investigated in this work. The effects of the porous layer on the rate of heat transfer and pressure drop are discussed and conditions for the optimal operating condition were suggested.

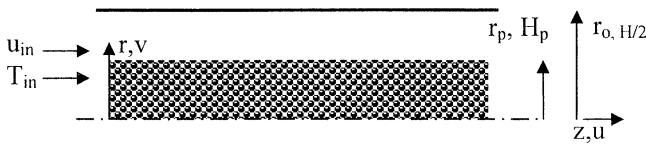


Fig. 1. Schematic diagram of the problem.

2. Problem definition

The schematic diagram of the problem is shown in Fig. 1. Laminar flow in a pipe or a channel partially filled with porous medium is considered. An air stream with a uniform velocity and temperature is considered at the inlet to the conduit. The wall temperature of the conduit is fixed, and is higher than the inlet temperature.

3. Governing equations

The flow is assumed to be two-dimensional, laminar and steady. It is also assumed that buoyancy effects are negligible. The governing equations may be written as:

continuity

$$\frac{\partial}{\partial z}(\rho u) + \frac{1}{r^n} \frac{\partial}{\partial r}(r^n \rho v) = 0; \tag{1}$$

z-momentum

$$\begin{aligned} \frac{\partial}{\partial z}(\rho uu) + \frac{1}{r^n} \frac{\partial}{\partial r}(r^n \rho vu) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left(\mu_e \frac{\partial u}{\partial z} \right) \\ + \frac{1}{r^n} \frac{\partial}{\partial r} \left(r^n \mu_e \frac{\partial u}{\partial r} \right) - f \frac{\mu u}{K} - f \frac{\rho F}{\sqrt{K}} |u|u; \end{aligned} \tag{2}$$

r-momentum

$$\begin{aligned} \frac{\partial}{\partial z}(\rho uv) + \frac{1}{r^n} \frac{\partial}{\partial r}(r^n \rho vv) = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial z} \left(\mu_e \frac{\partial v}{\partial z} \right) \\ + \frac{1}{r^n} \frac{\partial}{\partial r} \left(r^n \mu_e \frac{\partial v}{\partial r} \right) - f \frac{\mu v}{K} - f \frac{\rho F}{\sqrt{K}} |u|v - \frac{\mu v}{r^2} n. \end{aligned} \tag{3}$$

For flow in a pipe, n is set to unity and for flow in a channel, n is set to zero.

The parameter f is set to unity for flow in porous medium and to zero for flow in a region without porous material. Note that flux continuity (momentum and energy) is ensured by evaluating the harmonic mean values of the physical properties (viscosity, thermal conductivity) at the interface between the clear fluid and the fluid-saturated porous medium.

3.1. Energy equation

The energy equation, while neglecting viscous dissipation effect and heat generation, may be written as:

$$\begin{aligned} \frac{\partial}{\partial z}(\rho_e c_e u T) + \frac{1}{r^n} \frac{\partial}{\partial r}(\rho_e c_e r^n v T) \\ = \frac{\partial}{\partial z} \left(k_e \frac{\partial T}{\partial z} \right) + \frac{1}{r^n} \frac{\partial}{\partial r} \left(r^n k_e \frac{\partial T}{\partial r} \right), \end{aligned} \tag{4}$$

where k_e , c_e and ρ_e are the effective thermal conductivity, effective specific heat and effective density of the medium, respectively.

In modeling of the energy transport, it is assumed that the local thermal equilibrium exists between solid and fluid phases. Thermal equilibrium condition is adopted by Kaviany [7]. Also, previous work of the author [16] revealed that the thermal equilibrium assumption is valid as far as there is no heat released in the fluid phase (combustion, for instance) or in solid phase (catalytic effect, for instance). Moreover, a few tests done with two energy equation and it is found that the results are not that sensitive to the non-equilibrium condition for $Nu_l (h_v D^2 / k_f)$ of 100. For $Nu_l = 10$, which is very low practically, the difference was less than 1.0%. Therefore, it is assumed that thermal equilibrium is valid.

Axisymmetric boundary conditions are adopted at $r = 0$, i.e., $v = 0$ with the gradients of u and T in the r -direction set to zero. The v velocity component is set to zero, $u = u_{in}$ and $T = T_{in}$ at $z = 0$. For $z = L$, the gradients of the variables in the z -direction are set to zero. For $r = r_o$ and $0 < z < L$, no slip condition is assumed, i.e., $u = v = 0$ and $T = T_w$.

The above equations are nondimensionalized by using the inlet velocity, inlet temperature, and the pipe radius (or half height of the channel) as references to scale velocity components, temperature and length, respectively. Hence, the momentum and energy equations may be reformulated as:

Z-momentum

$$\begin{aligned} \frac{\partial}{\partial Z}(UU) + \frac{1}{R^n} \frac{\partial}{\partial R}(R^n VU) = -\frac{\partial P}{\partial Z} + \frac{\partial}{\partial Z} \left(\frac{\partial U}{\partial Z} \right) \\ + \frac{1}{R^n} \frac{\partial}{\partial R} \left(R^n \frac{\partial U}{\partial R} \right) - f \frac{U}{DaRe} - f \frac{F}{\sqrt{Da}} |U|u; \end{aligned} \tag{5}$$

R-momentum

$$\begin{aligned} \frac{\partial}{\partial Z}(UV) + \frac{1}{R^n} \frac{\partial}{\partial R}(R^n VV) = -\frac{\partial P}{\partial R} \\ + \frac{\partial}{\partial Z} \left(\frac{\partial V}{\partial Z} \right) + \frac{1}{R^n} \frac{\partial}{\partial R} \left(R^n \frac{\partial V}{\partial R} \right) \\ - f \frac{V}{DaRe} - f \frac{F}{\sqrt{Da}} |U|V - \frac{V}{ReR^2} n; \end{aligned} \tag{6}$$

and energy equation:

$$\begin{aligned} \frac{\partial}{\partial Z}(U\theta) + \frac{1}{R^n} \frac{\partial}{\partial R}(R^n V\theta) = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial Z^2} \right) \\ + \frac{1}{PeR^n} \frac{\partial}{\partial R} \left(R^n \frac{\partial \theta}{\partial R} \right), \end{aligned} \tag{7}$$

where Re , Da and Pe are Reynolds, Darcy and Peclet numbers, respectively.

3.2. Nusselt number calculations

The Nusselt number for a pipe can be calculated as:

$$Nu_D = \frac{2 \frac{\partial \theta}{\partial R}}{\theta_w - \theta_m}, \tag{8}$$

where θ_m stands for the fluid bulk temperature inside the pipe.

For flow in a channel, the Nusselt number is calculated as:

$$Nu_H = \frac{4 \frac{\partial \theta}{\partial R}}{\theta_w - \theta_m}. \quad (9)$$

The bulk temperature for flow in a pipe and in a channel can be calculated as:

$$\theta_m = \frac{\int_0^1 U \theta R^n dR}{\int_0^1 U R^n dR}. \quad (10)$$

4. Method of solution

4.1. Numerical solution

A control volume, finite-difference approach is used to solve the model equations with specified boundary conditions. The SIMPLER algorithm is employed to solve the equations in primitive variables. Central difference approximations are used to approximate the advection-diffusion terms, i.e., the scheme is second order accurate in space. The governing equations are converted into a system of algebraic equations through integration over each control volume. The algebraic equations are solved by a line-by-line iterative method. The method sweeps the domain of integration along the R - and Z -axis and uses the tri-diagonal matrix inversion algorithm to solve the system of equations. Velocity components are under-relaxed by a factor of 0.7. For most calculations, 4000 iterations are sufficient to get convergent solution for a 151×81 grid, and more iterations are needed for 201×101 . The criteria for convergence are to conserve mass, momentum, energy and species globally and locally, and to ensure convergence of pre-selected dependent variables to constant values within machine error.

In order to insure that the results are grid size independent, different meshes are tested namely 121×61 , 151×81 and 201×101 . The predicted results are compared for flow in a pipe without porous media. The developed velocity profile (parabolic) is compared with analytical solution and difference was not noticeable. The test was done for different Reynolds numbers and found that the developing length is inversely proportional of Reynolds number, which matches the analytical solution. Furthermore, the local Nusselt numbers along the pipe and the channel were well compared with analytical solutions. Since, results are available for a pipe and a channel fully filled with porous media, the predicted results were well compared with Nusselt number and velocity profile was uniform, except near the boundary. Quantitative comparison of the current results with available data will be presented in the following section. The calculations are performed by adopting nonuniform 151×61 grids in Z - and R -direction, respectively. Very fine grids are adopted near the boundaries. All the calculations were performed using double precision, which is necessary for the Nusselt number calculations.

4.2. Analytical solution

For fully developed conditions, the governing momentum (Darcy–Brinkman equation) and energy equations (axial heat diffusion is neglected) can be written as:

$$\frac{dP}{dZ} = \frac{1}{R^n} \frac{d}{dR} \left(R^n \frac{dU}{dR} \right) - f \frac{U}{DaRe}, \quad (11)$$

$$\frac{\partial}{\partial Z} (U\theta) = \frac{1}{PeR^n} \frac{\partial}{\partial R} \left(R^n \frac{\partial \theta}{\partial R} \right), \quad (12)$$

respectively. The mass conservation equation for fully developed flow is as follows:

$$1 = 2 \int_0^1 U R^n dR. \quad (13)$$

The above equation can be solved analytically to obtain the pressure gradient, velocity and temperature profiles.

For a partially filled conduit equation (11) should be integrated from $R = 0$ to $R = R_r$ with $f = 1$ (porous medium) and from $R = R_r$ to $R = 1$ with f set to zero. Also, Eq. (13) should be integrated in similar fashion.

5. Results and discussion

5.1. Fully filled conduits with porous material and inertia term

The Nusselt number for a fully developed, laminar, plug flow in a pipe with constant wall temperature is 5.78 [17]. The lower limit of the Nusselt number is 3.658 (for example see Oosthuizen and Naylor [18]), which is the Nusselt number for fully developed flow in a pipe without porous material. Therefore, it is expected that the plug flow assumption should not be valid for a pipe or a channel filled with high porous material. The present results, Fig. 2, show that the plug flow assumption is valid for $Da < 10^{-4}$. While increasing Darcy number (increasing permeability) the Nusselt number decreases monotonically and the rate of decrease depends on the model used. For Darcy–Brinkman model without the inertia term the effect of the presence of porous material becomes insignificant for $Da > 1.0$. Including inertia term (Forchheimer term) the flow becomes function of Reynolds number and inertia coefficient. The inertia term represents the nonlinearity of the pressure drop as the particle Reynolds number increases. The physics behind adding the inertia term to the Navier–Stokes equations is to account for the local drag force. The results indicated that the effect of inertia term decreases as Darcy number decreases and becomes insignificant for $Da < 10^{-4}$. The inertia coefficient is function of the topology of the porous media and it is difficult to correlate for a given porous matrix. More importantly, the results for $F = 1$ and $Re_r = 400$ showed that the difference between Nusselt

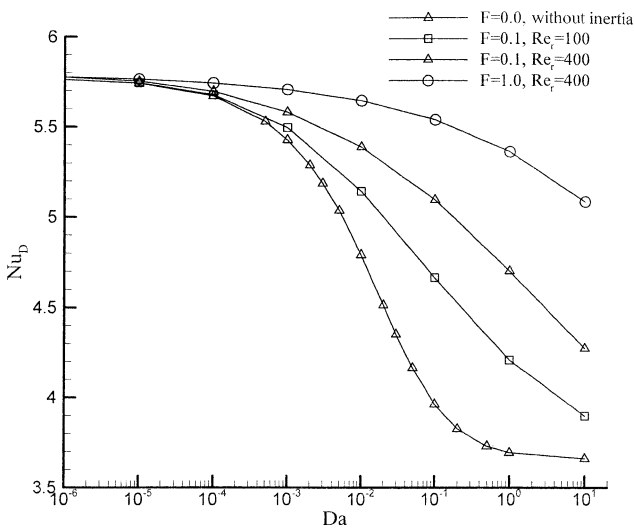


Fig. 2. Nusselt number as a function of Darcy number for fully developed pipe flow.

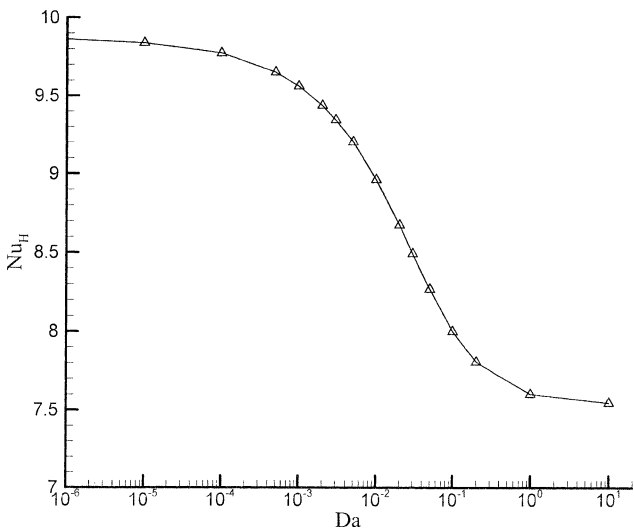


Fig. 3. Nusselt number as a function of Darcy number for fully developed channel flow.

number for $Da = 100$ ($Nu_D = 4.698$) and for flow in a pipe without porous material ($Nu_D = 3.658$) is large. Technically speaking, $Da = 100$ simulates flow in a pipe almost without porous material. Even for $F = 0.1$ and $Da = 100$, the Nusselt number was 3.944. It may be concluded that the inertia term should be modified for flow in high permeability medium or the coefficient F should be correlated properly. Since, it is not clear how the inertia term to be correlated, it's effect is neglected and assumed that $F = 0.0$ for further calculations.

The results for fully developed flow through a 2-D channel are illustrated in Fig. 3. As for the flow in a pipe, the plug flow assumption is not valid for $Da > 10^{-4}$. The Nusselt number asymptotically approaches 9.860, which the Nusselt number for the plug flow [18]. On the other hand the Nusselt number asymptotically approaches 7.541 for $Da >$

Table 1

Nusselt number for fully developed flow in pipes and channels

Darcy number	Nu_D for pipe	Nu_H for channel
10^{-6}	5.760	9.860
10^{-5}	5.740	9.838
10^{-4}	5.669	9.773
5×10^{-4}	5.530	9.651
10^{-3}	5.428	9.561
2×10^{-3}	5.289	9.437
3×10^{-3}	5.188	9.344
5×10^{-3}	5.036	9.203
10^{-2}	4.791	8.963
2×10^{-2}	4.515	8.675
3×10^{-2}	4.353	8.493
5×10^{-2}	4.165	8.268
10^{-1}	3.963	8.002
0.2	3.827	7.808
0.5	3.731	7.659
1.0	3.695	7.602
10.0	3.661	7.547
Without porous	3.658	7.541

1.0, which is the Nusselt number for the fully developed flow in a channel without porous medium [19].

Table 1, summarizes the results of Nusselt number as a function of Darcy number for the pipe and channel flows.

5.2. Partially filled conduits with porous medium

Nusselt number variation along the pipe as a function of Gz^{-1} number for $Da = 10^{-3}$, 10^{-4} , 10^{-5} and 10^{-6} are shown in Fig. 4(a), (b), (c) and (d), respectively, and for different porous radius ratios. In general, the Nusselt number (rate of heat transfer) increases as the porous radius increases. This is due to the fact that the fluid escapes from the high resistance region (porous region) to the outer region; accordingly the boundary layer thickness decreases. The flow mostly channels between the porous layer and the pipe wall. Such a channeling effect take place for $R_r \leq 0.8$, further increase in porous diameter decreases the clear fluid gap and flow experience high resistance, where the flow does not have preference region to flow, therefore, the Nusselt number decreases. As far as heat transfer concerns, the optimal value for R_r is about 0.8. The developing length is not significantly changes with Da number, except for $R_r = 0.8$. For $R_r = 0.8$, as Da decreases the developing length decreases.

Fig. 5(a) and (b) show isotherms for different radius ratios and for $Da = 10^{-3}$ and $Da = 10^{-4}$, respectively. The interesting thing that can be learned from the figures is that, as the porous layer thickness increases, the thermal developing length decreases, which supports the results presented in Fig. 4. For instance, the fully developed lengths for $R_r = 0.4$ and $R_r = 0.6$ are about 35 and 24, respectively, for $Da = 10^{-3}$ and are about 33 and 22.5 for $Da = 10^{-4}$. This suggests that adding porous layer beyond the mentioned lengths has no effect on the rate of heat transfer.

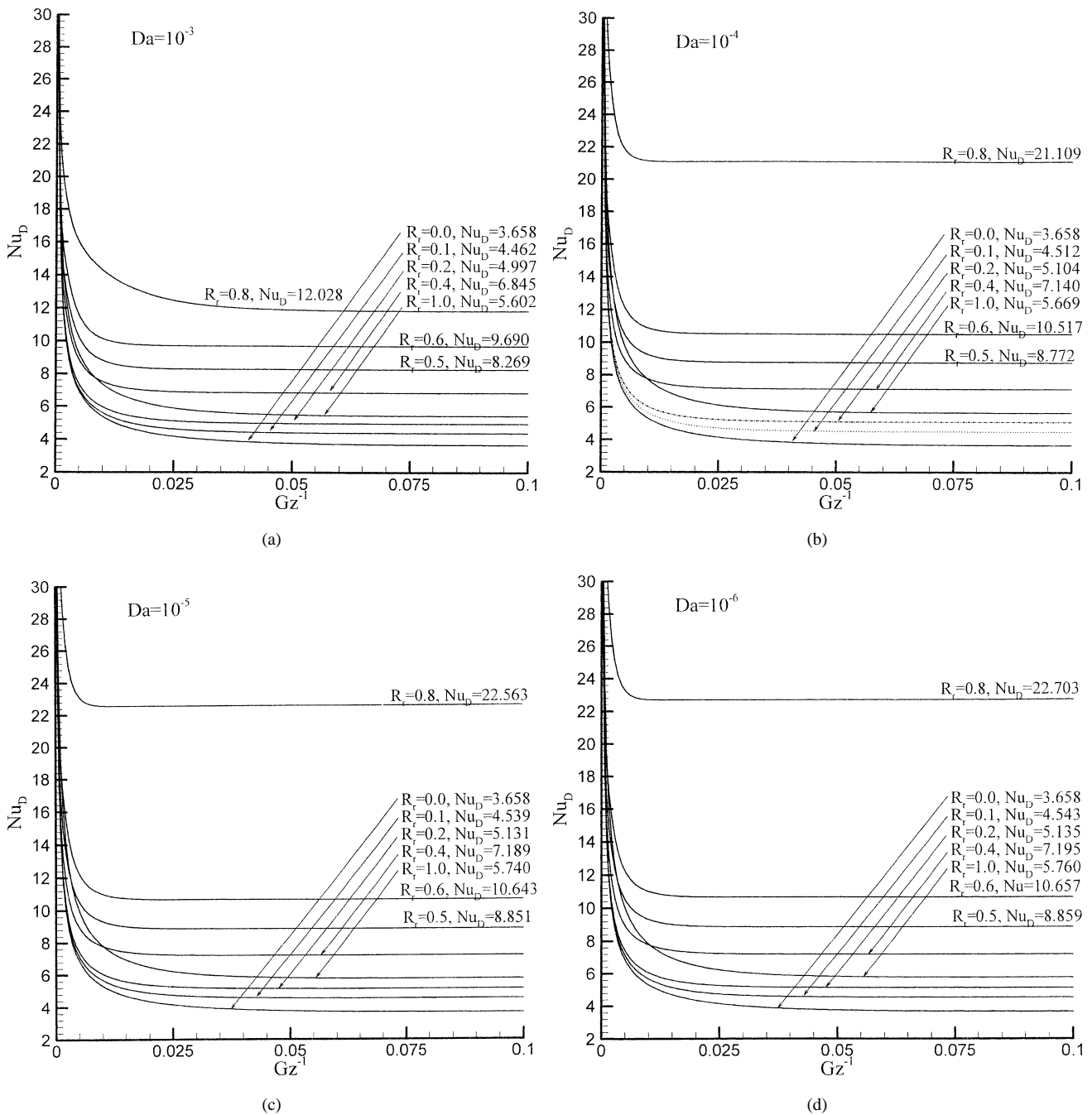


Fig. 4. Nusselt number as a function of Graetz number for different porous material radius ratio: (a) $Da = 10^{-3}$, (b) $Da = 10^{-4}$, (c) $Da = 10^{-5}$ and (d) $Da = 10^{-6}$.

Therefore, adding porous media partially into the a conduit can reduce developing length by 50% or more.

Fig. 6(a) and (b) illustrate the fully developed velocity profiles for $Da = 10^{-3}$ and $Da = 10^{-6}$, respectively, and for different porous radius ratios. For $Da = 10^{-6}$, the flow mainly channels between the porous medium and pipe wall for $R_r < 0.8$, where the flow through the porous medium is negligible. Since, the flow channels between the porous matrix and pipe wall, therefore the Nusselt number should equal to the Nusselt for annular flow. For instance, the Nu

for annular flows are 4.56 and 8.86 for radius ratios of 0.1 and 0.5, respectively, which is very well compared with present results presented in Fig. 4(d). For $R_r = 1.0$, the pipe is completely filled with porous medium and plug flow assumption is valid. For $Da = 10^{-3}$, the problem is different due to moderate permeability of the porous medium. The flow rate in the porous medium decreases as R_r increases from 0.1 to 0.4, then the flow rate through the porous increases as the R_r increases due to the hydraulic resistance increment in the channeling region. Also, the plug flow

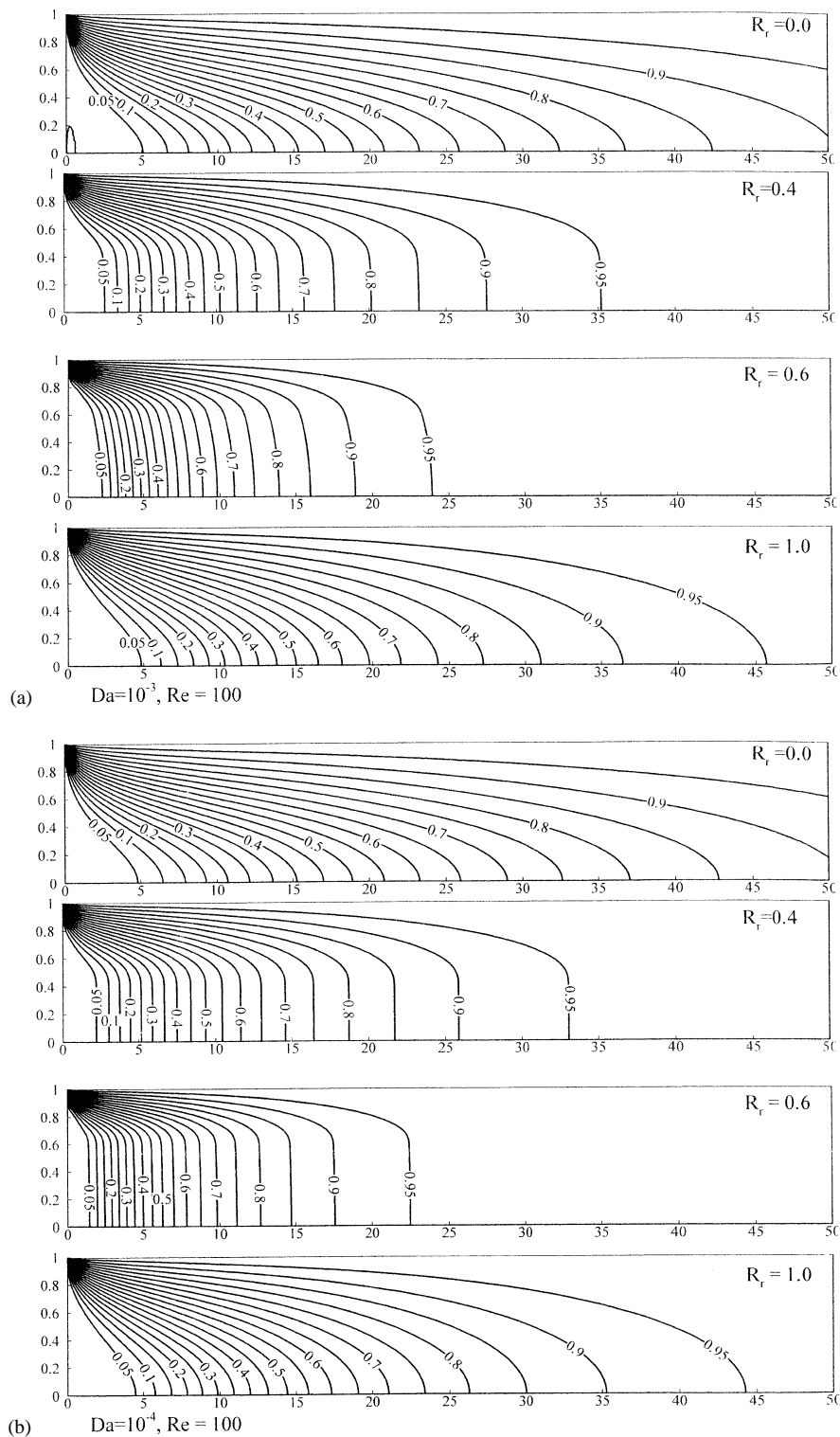
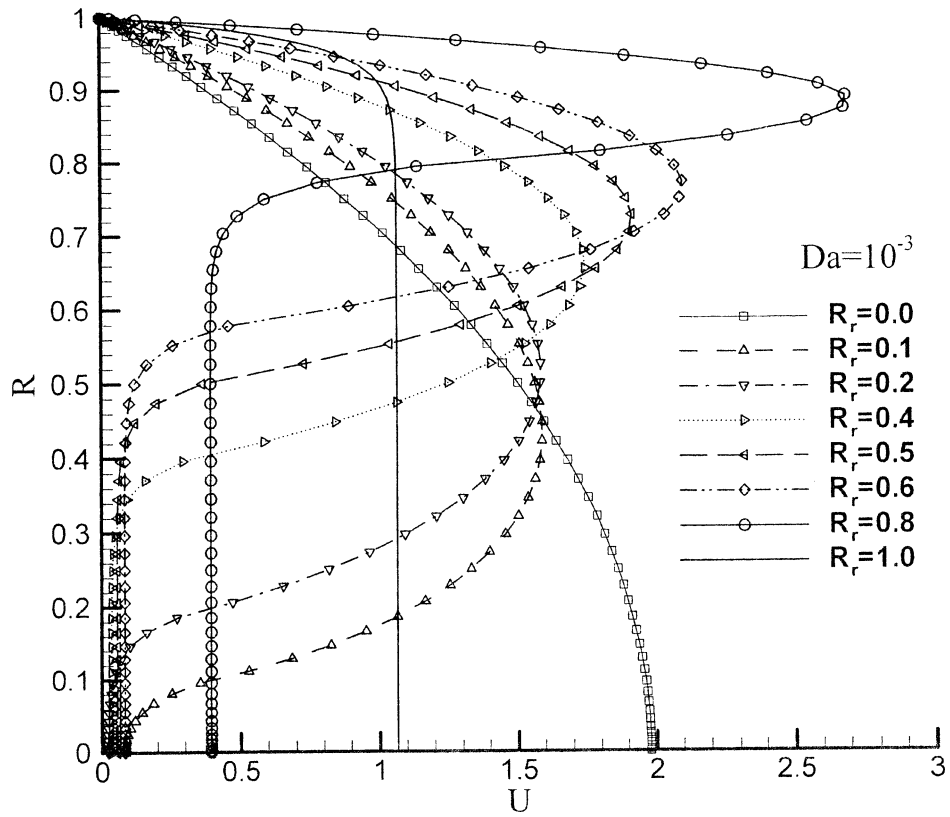


Fig. 5. Isotherms for a flow in a pipe with different radius ratios: (a) $Da = 10^{-3}$ and (b) $Da = 10^{-4}$.

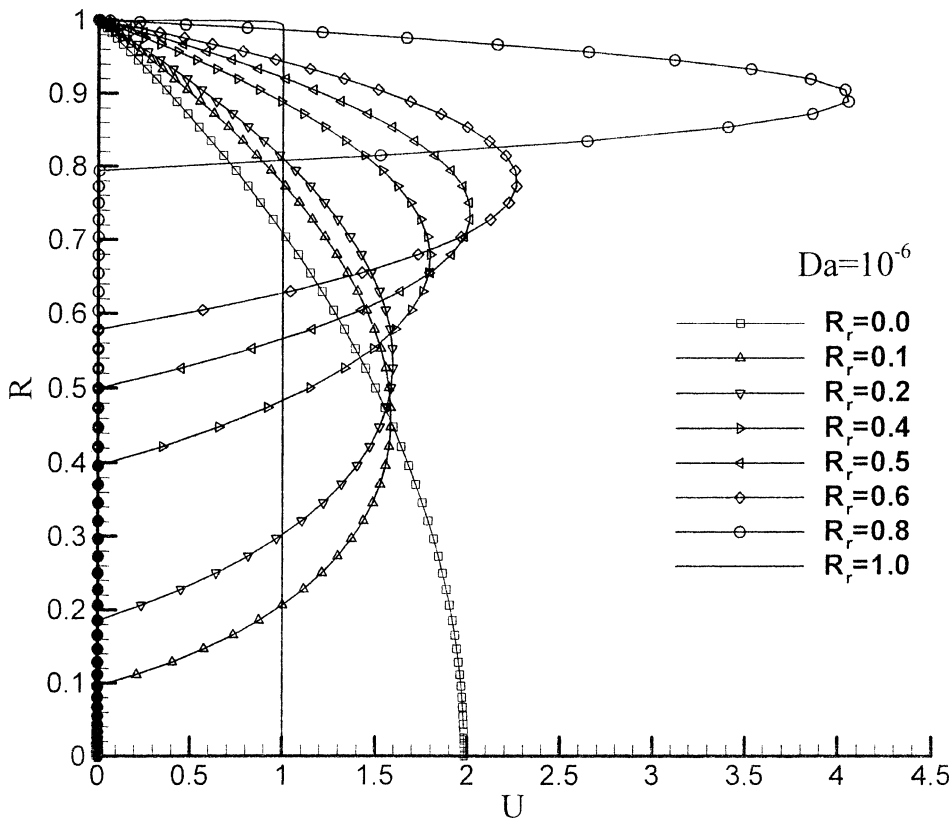
assumption is not valid for a flow in pipe filled with porous material.

Heat exchanger analysis can not be completed without pressure drop analysis. For fully developed flows, the pressure gradient as a function of R_r is represented in Fig. 7. For R_r less than about 0.6, the effect of Darcy number on the

pressure drop is not that significant. The pressure drop for a fully filled pipe with porous material is much higher than that for a pipe partially filled with porous material. Since, the pressure drop (Fig. 7) and Nusselt number (Fig. 4) are not strong function of Da for $R_r < 0.6$, therefore it may be that R_r of about 0.6 is a good value to use in enhancing heat



(a)



(b)

Fig. 6. Fully developed velocity profiles for different porous material radius ratio: (a) $Da = 10^{-3}$ and (b) $Da = 10^{-6}$.

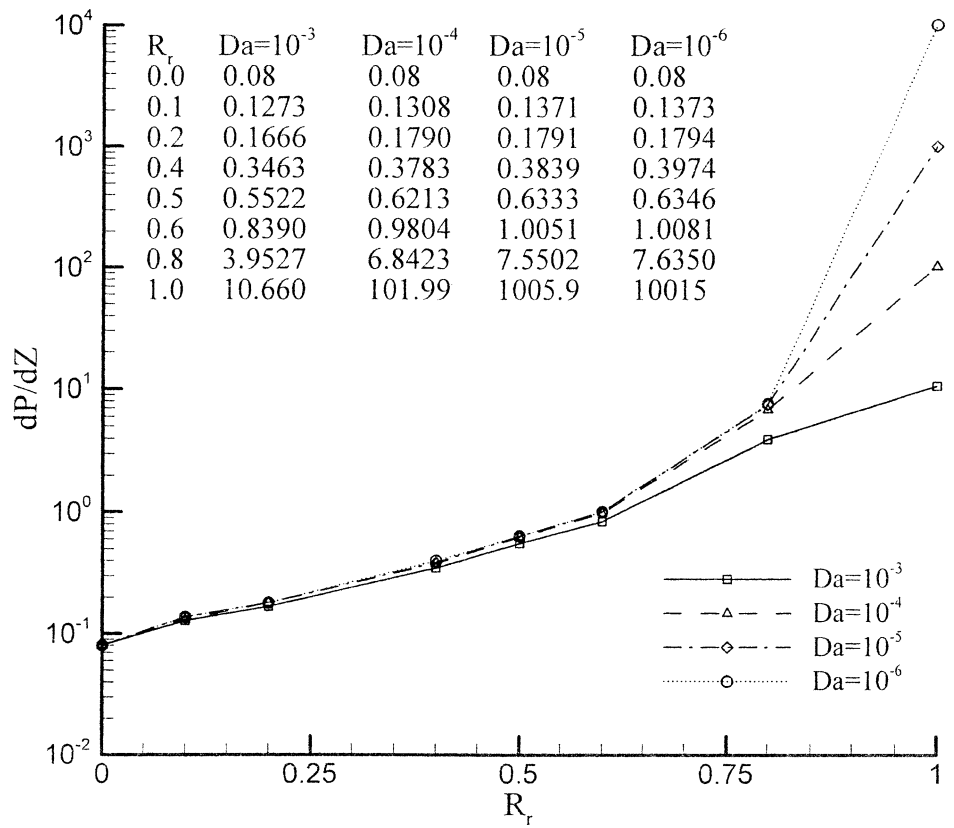


Fig. 7. Pressure gradients for fully developed pipe flow as a function of porous radius ratio and for a range of Darcy number.

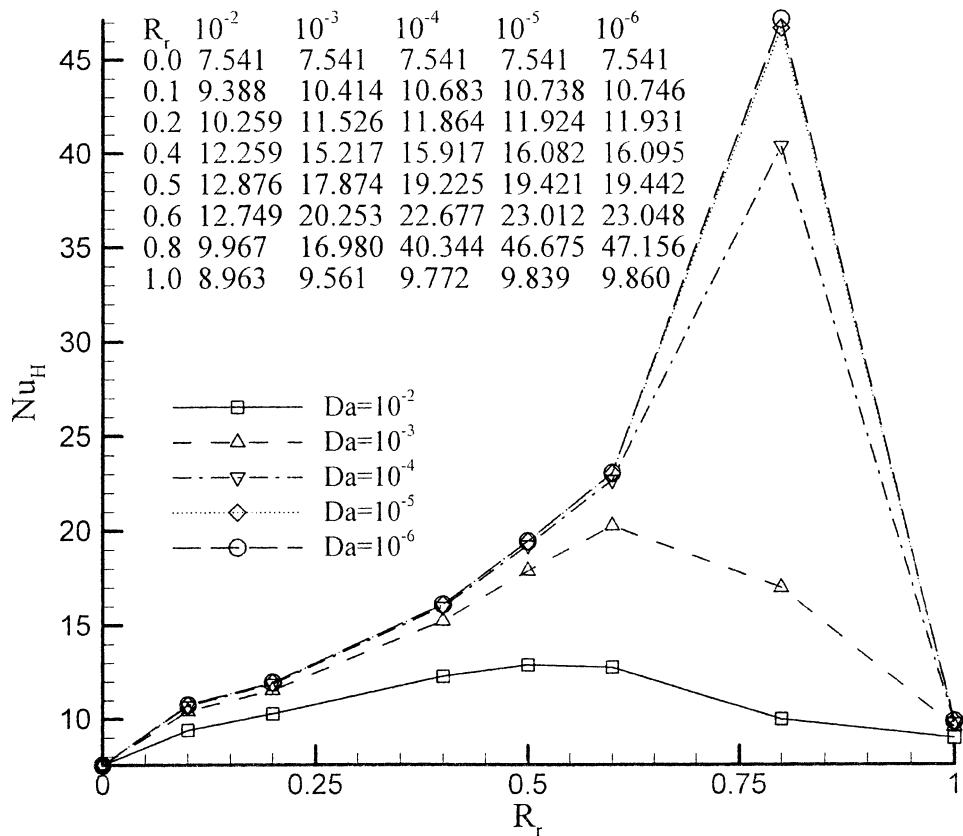


Fig. 8. Nusselt number for fully developed channel flow as a function of porous layer thickness and for a range of Darcy number.

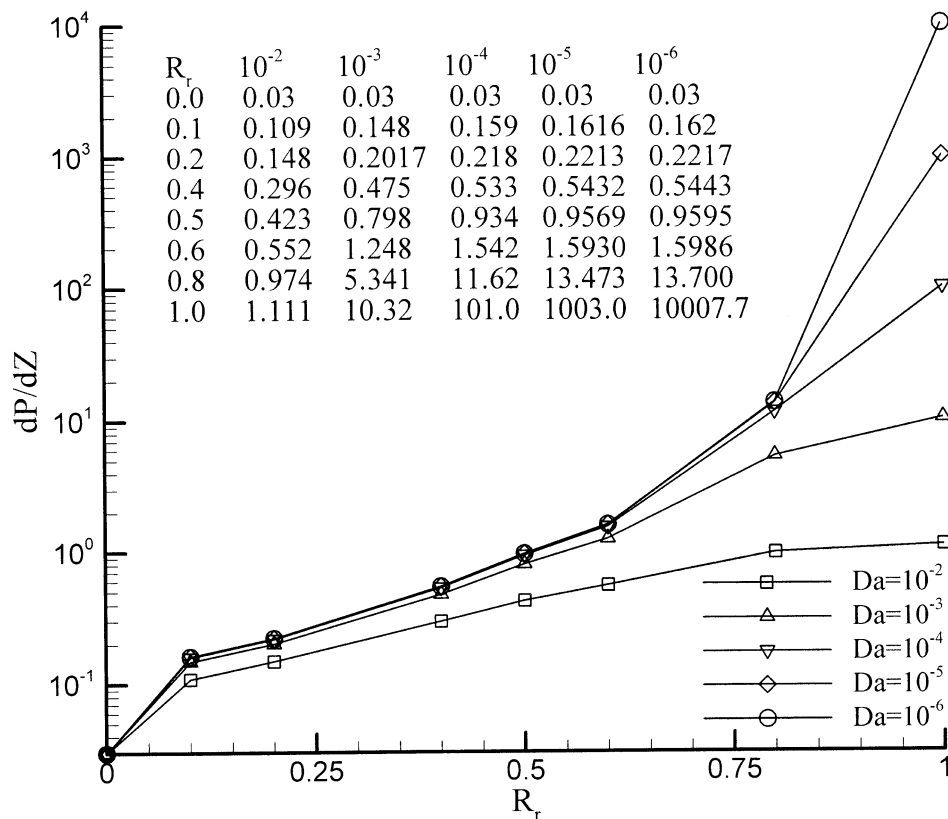


Fig. 9. Pressure gradients for fully developed channel flow as a function of porous layer thickness and for a range of Darcy number.

transfer in heat exchangers. The results for $R_r = 0.8$, yields high rate of heat transfer, almost double compared for that of $R_r = 0.6$, but the pressure drops increases by factor of about 7.0.

Similar calculations were performed for channel and results of Nusselt number for fully developed flow are represented in Fig. 8. For $Da \geq 10^{-3}$, the maximum value of Nusselt number take places for R_r of about 0.6. While for $Da \leq 10^{-4}$, the maximum rate of heat transfer (Nu) increases as R_r increases to about 0.8, then decreases as R_r increases. As far as the pressure drop concerns, Fig. 9 shows the variation of pressure gradient as a function of R_r and for different Darcy numbers. Similar to the results of the pipe flow, the pressure drop is not a strong function of the Darcy number for $Da < 10^{-2}$ and for $R_r \leq 0.6$. The pressure drop drastically increases for fully filled channel and for $Da < 10^{-4}$.

6. Conclusions

Heat transfer enhancements and pressure drops for flow in pipes and channels inserted with porous materials are presented, for a range of Darcy number and R_r . It is found that the plug flow assumption is not valid for $Da > 10^{-4}$, where the permeability of the medium is high. Inertia term has significant effect on the Nusselt, but it is not clear that the correlation of the inertia term is valid for a highly porous

layer. The flow developing length is not strong function of Darcy number. Filling a conduit partially with porous media can reduce the thermally developing length by 50% or more. The rate of heat transfer increases by adding porous material into the core of a conduit. As far as the pressure drop concern, the optimum porous thickness or radius ratio is about 0.6, where the heat transfer can be enhanced with a reasonable pressure drop.

The effect of the inertia term needed to be justified experimentally or analytically. The current model of Forchheimer term does not produce asymptotic results as the Darcy number increase. Constant heat flux boundary condition will be analyzed and represented in the part II of the paper.

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